A Gamut Preserving Color Image Quantization

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Abstract

We propose a new approach for color image quantization which preserves the shape of the color gamut of the studied image. Quantization consists to find a set of color representative of the color distribution of the image. We are looking here for an optimal LUT (look up table) which contains informations on the image's gamut and on the color distribution of this image. The main motivation of this work is to control the reproduction of color images on different output devices in order to have the same color feeling, coupling intrinsic informations on the image gamut and output device calibration. We have developped a color quantization algorithm based on an image dependant sampling of the CIELAB color space. This approach overcomes classical approaches.

1. Introduction

The main motivation of this work is to control the reproduction of color images on any devices in order to have the same color feeling. Most of management methods are based on the use of parametric models or LUTs which are determined during the characterization step to calibrate output devices. Most of output devices are limited by their color gamut, so they can display only a small amount of colors in comparison to the spectrum locus. In order to reduce the color differences between two devices, the use of a gamut mapping function is necessary. Such a function enables to map the color gamut of the input device into the color gamut of the reproduction device [1]. This paper focuses on the research of a specific LUT which both circumscribes the color gamut of the studied image and samples the color distribution of this image. This LUT would be used to find a model which will allow to estimate, with a strong reliability, the color distribution of the input image. In order to optimize the model determination (i.e. to minimize the errors of interpolation), the LUT computation should be done in a perceptually uniform color space, such as CIELAB, because equal Euclidean distances in this color space represent approximately equal perceived differences in appearance.

Most of quantization methods are based on the minimization of quantization errors but other requirements need to be taken into account [3, 4, 5]. The problem with such methods is that the shape of the color gamut is not taken into account, and suffers distortions. We present here an algorithm which settles this problem. In section 2 we present our algorithm. In section 3, some results are given. The quality of the quantization overcomes existing methods in both appearence of quantized image and preservation of objective informations such as color gamut's shape.

2. The algorithm

The color quantization technique that we propose is based on six steps and a split and merges strategy. The first step consists on a uniform sampling of L*a*b* color space based on a sphere packing technique constraining each sphere's center to be at equal distance from its neighbors so that all neighbors of one center form a Johnson polyhedron [6]. Each sample of the grid is then surrounded by 12 equidistant samples. The second step consists on removing all samples which do not belong to the gamut of the L*a*b* color space. The third consists on over-sampling the L*a*b* color space when the color distribution of the image around a sample is not correctly represented by only one sample. The main advantage of this process is that it improves the discretization accuracy where it is necessary without increasing computational time too much. The forth consists on sub-sampling the L*a*b* color space when the color distribution around a sample is empty. The fifth consists on the selection of the 2^n most representative color samples among those previously defined. The latest consists on a refinement of the set of color samples in order to make it more representative of the color image gamut.

2.1. Uniform sampling of CIELAB color space

To sample uniformly the L*a*b* color space we used a hexagonal based grid. The sampling is parameterized by the distance d_{ref} between two samples. The smaller the d_{ref} value the finer the sampling of L*a*b* space. Let be $\{\vec{P}_j\}$ the set of color selected by the uniform sampling process. Then, for each color $\vec{C}_i$ of the image studied, the closest sample is defined by $\arg\min_{j=0,\ldots K-1} \Delta E^*_{ab}\left(\vec{C}_i, \vec{P}_j\right)$. In the worst case, the maximal error is equal to $d_{ref}/\sqrt{2}$, i.e. to the isobarycentre of a regular pyramid defined by d_{ref}.

2.2. Over-sampling of CIELAB color space

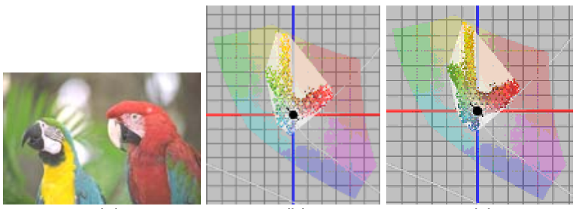
One way to reduce the mean quantization error is to increase the number of samples around samples for which the mean error is too high. To reduce the mean error, we have fixed the following constraint: *If* $average_{\{\vec{C}_i/\vec{P}_j\}} \Delta E^*_{ab}\left(\vec{C}_i, \vec{P}_j\right) > 25\%\ d_{ref}/\sqrt{2}$ *then sub-sample the hexagon centered on* $\vec{P}_j$, where $\{\vec{C}_i/\vec{P}_j\}$ represents the set of color $\vec{C}_i$ for which the closest color in color palette is the color $\vec{P}_j$. The interest of this over-sampling is that it breaks the regularity of the uniform sampling scheme and reduces computational time.

As example to sample the L*a*b* color space with a precision of $d_{ref} = 3$, we have developed a process which firstly uniformly samples this color space with a precision of $d_{ref} = 9$, next over-samples this color space with a precision of $d_{ref}/3 = 3$. Several studies had shown that under a precision of $d_{ref} = 2$ the errors of quantization are imperceptible for HSV and that with a precision of $d_{ref} = 3$ the errors become perceptible but are not annoying. So a good compromise between precision and computing time can be obtained with $d_{ref}/3 = 3$ or $d_{ref}/3 = 2$.

In order to reduce the palette size without increasing the quantization error, we have used the constraint *If* $card\ \{\vec{C}_i/\vec{P}_j\} = 0$ *then remove* $\vec{P}_j$ *of the color palette.* Where $card\ \{\vec{C}_i/\vec{P}_j\}$ represents the number of colors of the original image closer of $\vec{P}_j$ than any other color sample This constraint enables to remove all useless samples of the color palette.

2.3. Selection of representative color samples

Let $Density_1(P_j) = \dfrac{card\ \{\vec{C}_i/\vec{P}_j\}}{\max_{k=0,\ldots K-1} card\ \{\vec{C}_i/\vec{P}_k\}}$ be the probability of density of $\vec{P}_j$ defined in function of the number of colors of the original image distributed around each sample. Note that most gamut mapping algorithms use this density criterion [1]. Let $Density_2(P_j) = \dfrac{1 - average_{\{\vec{C}_i/\vec{P}_j\}} \Delta E^*_{ab}\left(\vec{C}_i, \vec{P}_j\right) / \Delta E^* \max\{\vec{P}_j\}}{\max_{k=0,\ldots K-1}\left(1 - average_{\{\vec{C}_i/\vec{P}_k\}} \Delta E^*_{ab}\left(\vec{C}_i, \vec{P}_k\right) / \Delta E^* \max\{\vec{P}_k\}\right)}$ be the relative probability density of the sample $\vec{P}_j$ defined in function of the density distribution of other samples. Let $Density_3(P_j) = \frac{1}{n}\sum_{i=0}^{n} SD\ \left(\vec{C}_i/\vec{P}_j\right)$ be the relative spatial density of sample $\vec{P}_j$. Considering that the color of a pixel corresponding to a low frequency shift is perceptually more important for an observer than those of a high frequency shift [7], this density criterion is used to weight the spatial frequency shifts of all colors linked to each sample. We have combined these three density functions to order samples from the most representative to the less. The idea is to reduce the number of samples to the most representative ones. To do that we have defined the following fully image dependant function: $Density(P_j) = Density_2(P_j) * Density_1(P_j) + (1 - Density_2(P_j)) * Density_3(P_j)$. Note that most of quantization algorithms use only the first density function, or give more weight to the first density function than to the third [2].



(a) (b) (c)

Figure 1. Example of color palettes in L*a*b* color space. (a) Image Parrots, (b) sampling of L*a*b* with $d_{ref}/3 = 3$, (c) sampling of L*a*b* with $d_{ref}/3 = 2$

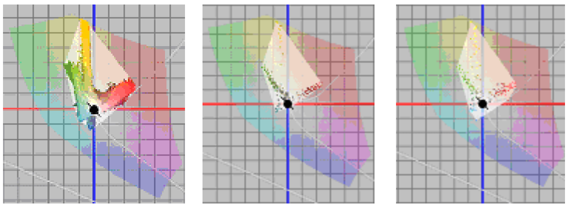
Next, we have applied the following algorithm to reduce the size of color palette to a given number of representative csamples: *While* $N > 2^n$ *remove the color sample* $\vec{P}_k$ *defined by:* $\operatorname{argmin}_{j=0,\ldots K-1} Density(P_j)$; *then* $N = N - 1$; *then redistribute image's colors which have lost their leader sample.* Let be $\{\vec{P}_j\}$ the new color palette defined by this process. As example, see Fig. 1, Table 1 and Fig. 2. Let us note on Fig. 2 that the gamut of the color palette has been unfortunately reduced by selection of the most representative colors (see (b) and (d)) with regard to the color gamut of all color samples (see (c) and (e)). To compensate this drawback, it is necessary to use a complementary criterion based on the Voronoi's partition. This criterion is used *a posteriori* to refine the color palette.

Table 1. Quantization errors (mean ΔE) *before and **after selection of the 256 most representative colors.

Image Parrots sampled with	Number of samples*	Quantization errors (* / **)
$d_{ref}/3=2$	5717	1.33 / 2.23
$d_{ref}/3=3$	1759	1.88 / 2.37

2.4. Refinement of the set of selected samples

Let $Compacity\ (P_j)=\dfrac{V\left(S\left(\vec{P}_j,R_j\right)\right)/V\left(C\left(\vec{P}_j\right)\right)}{\max_{k=0,\dots,K-1}\left(V\left(S\left(\vec{P}_k,R_k\right)\right)\ /\ V\left(C\left(\vec{P}_k\right)\right)\right)}$ be the compactness of the sample $\vec{P}_j$ defined in function of the density of colors distribution of the original image represented by this sample. Where: $V\left(S\left(\vec{P}_j,R_j\right)\right)$ represents the volume of the sphere $S\left(\vec{P}_j,R_j\right)$ centered on $\vec{P}_j$ of radius $R_j = mean_{\{\vec{C}_i/\vec{P}_j\}}\ \Delta E^*_{ab}\left(\vec{C}_i,\vec{P}_j\right)$; $V\left(C\left(\vec{P}_j\right)\right)$ represents the volume of the Voronoï's cell $C\left(\vec{P}_j\right)$ centered on $\vec{P}_j$. The higher this compactness estimation the more the weight of the studied color sample is important for the color palette of $\{\vec{P}_j\}$. Then, if two color samples of $\{\vec{P}_j\}$ are less distant than $d_{ref}/3$ we consider than one of these two samples can be withdrawn of the set $\{\vec{P}_j\}$ and replaced by another sample of $\{\vec{P}_j\}$ more relevant in regards to the color gamut of the image studied. Let $Eccentricity(\vec{P}_j)=10-\left\lfloor 10\ \times\ \dfrac{d\left(\vec{P}_j\ ,CH_I\right)}{d_{\max}/2}\right\rfloor$ be the distance of the sample $\vec{P}_j$ to the convex hull of the gamut of $\{\vec{P}_j\}$ scaled on 10 values. Where: $d\left(\vec{P}_j\ ,CH_I\right)$ represents the color distance of sample $\vec{P}_j$ to the convex hull of the gamut of $\{\vec{P}_j\}$; $d_{\max}(CH_I)$ represents the maximal diameter of the convex hull of the gamut of $\{\vec{P}_j\}$; $\lfloor x \rfloor$ the integer value of x. The higher this eccentricity the higher the studied sample plays an important role in the computation of the shape and of the size of the color gamut of $\{\vec{P}_j\}$. Then, we applied the following algorithm: *(1) order $\{\vec{P}_j\}$ in a queue firstly from upper to lower eccentricity value, next from upper to lower density value according to $Density_2(P_j)*Density_1(P_j)$. (2) if two samples $\vec{P}_1$ and $\vec{P}_2$ of $\{\vec{P}_j\}$ are less distant than $d_{ref}/3$ (or less distant than 2 if $d_{ref}\leq 6$) and $Compacity\ (P_1) > Compacity\ (P_2)$ then withdraw $\vec{P}_1$ of $\{\vec{P}_j\}$ and replace it by the sample at the top of the pile. Re-iterate (2).* With such a process we preserve better the shape and the size of the gamut of the palette with regard to the shape and the size of the gamut of the studied image. Likewise, the maximal error decreases but with depends of the average error. As example, see the results given in Fig. 2 and Table 2.



(a) (b) (c)

Vol. (gamut of Q1)= 87858.3
Vol. (gamut Q1)/Vol.(gamut I)=43%
$mean\left(\Delta E^*_{ab}\right) = 2.23$
$\sigma\left(\Delta E^*_{ab}\right) = 2.18$ $\max\left(\Delta E^*_{ab}\right) = 44.08$

(d)

Vol. (gamut of Q2)= 154035
Vol.(gamut Q2)/Vol.(gamut I) = 75%
$mean\left(\Delta E^*_{ab}\right) = 2.40$
$\sigma\left(\Delta E^*_{ab}\right) = 2.23$ $\max\left(\Delta E^*_{ab}\right) = 14.60$

(e)

Figure 2. Image Parrots sampled with 256 color and $d_{ref}=6$ before and after refinement of the color palette. Gamuts of: Parrots (a), Q1 (b) and Q2 (c).

Table 2. Parrot image quality of quantized image (Q) before and after refinement of the color palette by the compactness criterion.

Image Parrots (I) Volume(gamutI) = 206593	Image quantized (Q) Gamut increase (before/after)	Image quantized (Q) Average error increase (before/after)	• $mean\left(\Delta E^*_{ab}\right)$ • $\sigma\left(\Delta E^*_{ab}\right)$ • $\max\left(\Delta E^*_{ab}\right)$ (after)	• MSE x10^{-4} • RMSE x10^{-2} • PSNR (after)
$d_{ref}=9$ and 64 samples	2 points (31% / 33%)	3.45 to 3.61	• 3.61 • 3.40 • 45.51	• 16.59 • 4.07 • 27.80
$d_{ref}=6$ and 64 samples	2 points (29% / 31%)	3.48 to 3.49	• 3.48 • 3.40 • 45.57	• 17.47 • 4.18 • 27.57
$d_{ref}=9$ and 128 samples	18 points (35% / 53%)	2.86 to 3.24	• 3.24 • 2.79 • 45.57	• 11.36 • 3.37 • 29.45
$d_{ref}=6$ and 128 samples	1 points (37% / 38%)	2.72 to 2.76	• 2.76 • 2.66 • 46.69	• 10.49 • 3.24 • 29.79
$d_{ref}=9$ and 256 samples	12 points (48% / 60%)	2.37 to 3.22	• 3.22 • 2.30 • 34.22	• 10.41 • 3.22 • 29.82
$d_{ref}=6$ and 256 samples	32 points (43% / 75%)	2.23 to 2.40	• **2.40** • **2.23** • **14.60**	• **6.86** • **2.61** • **31.63**

3. Results and discussion

To evaluate the impact of the d_{ref} distance on the quantization results, we have compared the results obtained by our algorithm with the results of the center-cut, the median-cut, the variance-based, the octree algorithms. We have also analysed the impact of the refinement method based on the compactness criterion. The results show (see Table 3) that the quality of the quantization of image Lena in 256 colors is better with our algorithm (with $d_{ref} = 6$) than with any other ones. These results show also that the refinement of the color gamut by the compactness criterion does not necessarily decrease the quality of the quantization. For image Lena, the quality of quantization is slightly worse after refinement than before. Nevertheless the quality after refinement is better than those of the other ones. We have also compared the size of the color gamut obtained by our algorithm with those given by the other ones (Fig. 3). For image Lena quantized in 256 colors, the color gamut which maps better the color gamut of the original image is those given by our algorithm (with $d_{ref} = 6$).

Table 3. Lenna Quantized in 256 colors using several quantization algorithm.

	MSE	PSNR	$mean(\Delta E^*_{ab})$
Center-cut	0.004	23.63	5.59
Median-cut	0.007	21.49	6.38
Variance-based	0.018	17.39	7.49
Octree	0.020	16.82	8.33
Our algorithm with $d_{ref} = 9$ and before refinement	0.00067	31.73	2.112
Our algorithm with $d_{ref} = 9$ and after refinement	0.002	26.37	4.27
Our algorithm with $d_{ref} = 6$ and before refinement	**0.00049**	**33.04**	**1.78**
Our algorithm with $d_{ref} = 6$ and after refinement	0.0006	32.05	2.18

4. Conclusion

The purpose of this paper was to build a color image quantization scheme, based on an image dependent color gamut sampling of the L*a*b* color space, which preserve the color gamut shape of the image. Results obtained shown that the newly proposed scheme outperforms existing quantization algorithms, in particular it better preserves the gamut of the image studied and the color distribution of this image, and gives very good results in terms of image quality. We have also shown that the increase of number of samples enables to better preserve the gamut of the image studied especially when d_{ref} is low. This increase is done with depend of computing time but has no influence on precision of sampling. We have computed from different image sets (e.g. the Kodak image set) that in a general way $d_{ref}/3 = 3$ or $d_{ref}/3 = 2$ is a good compromise between precision and computing, and that the choice of d_{ref} value is also image dependent.

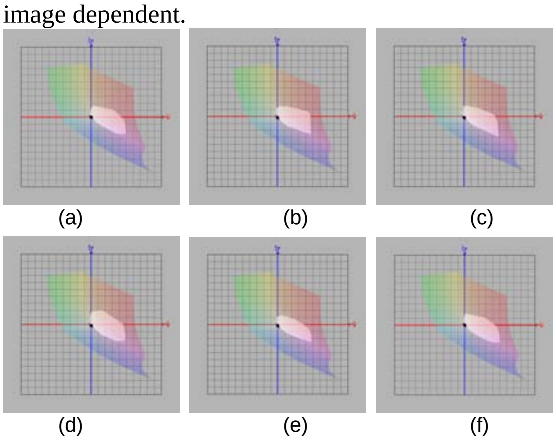


Figure 3. Comparison of image Lena's color gamut (convex hull) quantized in 256 colors in the a*b* color diagram. (a) Image Lena, (b) Median-cut, (c) Variance-based, (d) Octree, (e) Bing et al., (f) Our algorithm with $d_{ref} = 6$ after gamut refinement.

5. References

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